Exercise 3

A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.

(b) What is the velocity after 1 second?

(c) When is the particle at rest?

(d) When is the particle moving in the positive direction?

(e) Find the total distance traveled during the first 6 seconds.

(f) Draw a diagram like Figure 2 to illustrate the motion of the particle.

(g) Find the acceleration at time t and after 1 second.

(h) Graph the position, velocity, and acceleration functions for $0 \leq t \leq 6$.

(i) When is the particle speeding up? When is it slowing down?

$$f(t) = \sin(\pi t/2)$$

Solution

Part (a)

To find the velocity, take the derivative of the position function.

$$\begin{aligned}
v(t) &= \frac{ds}{dt} \\
&= \frac{d}{dt} \sin \frac{\pi t}{2} \\
&= \cos \frac{\pi t}{2} \cdot \frac{d}{dt}\left(\frac{\pi t}{2}\right) \\
&= \cos \frac{\pi t}{2} \cdot \left(\frac{\pi}{2}\right) \\
&= \frac{\pi}{2} \cos \frac{\pi t}{2}
\end{aligned}$$

Part (b)

The velocity after 1 second has elapsed is

$$v(1) = \frac{\pi}{2} \cos \frac{\pi}{2} = 0 \, \frac{\text{feet}}{\text{second}}.$$

Part (c)

To find when the particle is at rest, set the velocity function equal to zero and solve the equation for t.

$$v(t) = 0$$

$$\frac{\pi}{2}\cos\frac{\pi t}{2} = 0$$

$$\cos\frac{\pi t}{2} = 0$$

$$\frac{\pi t}{2} = \frac{1}{2}(2n-1)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

$$t = 2n - 1$$

Since $0 \le t \le 6$, the particle is at rest when $t = 1$, $t = 3$, and $t = 5$.

Part (d)

To find when the particle is moving in the positive direction, find what values of t satisfy $v(t) > 0$.

$$v(t) > 0$$

$$\frac{\pi}{2}\cos\frac{\pi t}{2} > 0$$

$$\cos\frac{\pi t}{2} > 0$$

Note that a cosine curve is positive from 0 to $\pi/2$ and from $3\pi/2$ to 2π.

$$0 \le \frac{\pi t}{2} < \frac{\pi}{2} \quad \text{or} \quad \frac{3\pi}{2} < \frac{\pi t}{2} < \frac{5\pi}{2}$$

$$0 \le t < 1 \quad \text{or} \quad 3 < t < 5$$

Therefore, the particle is moving in the positive direction for $[0, 1) \cup (3, 5)$.

Part (e)

The distance travelled in $0 \le t < 1$ is

$$|s(1) - s(0)| = \left|\sin\frac{\pi}{2} - \sin\frac{\pi(0)}{2}\right| = 1,$$

the distance travelled in $1 < t < 3$ is

$$|s(3) - s(1)| = \left|\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right| = 2,$$

the distance travelled in $3 < t < 5$ is

$$|s(5) - s(3)| = \left|\sin\frac{5\pi}{2} - \sin\frac{3\pi}{2}\right| = 2,$$

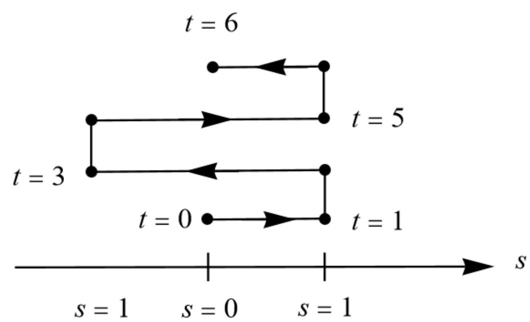
and the distance travelled in $5 < t < 6$ is

$$|s(6) - s(5)| = \left|\sin\frac{6\pi}{2} - \sin\frac{5\pi}{2}\right| = 1.$$

Consequently, the total distance travelled in $0 \le t \le 6$ is $1 + 2 + 2 + 1 = 6$ feet.

Part (f)

Below is an illustration of the particle's motion from $t = 0$ to $t = 6$.



Part (g)

Calculate the derivative of the velocity to get the acceleration.

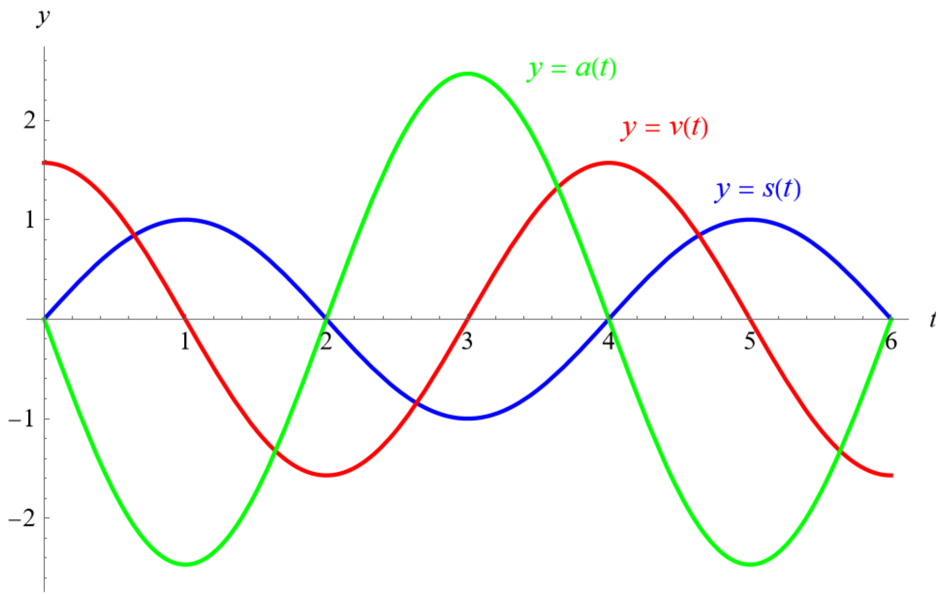
$$\begin{aligned} a(t) &= \frac{dv}{dt} \\ &= \frac{d}{dt}\left(\frac{\pi}{2}\cos\frac{\pi t}{2}\right) \\ &= \frac{\pi}{2}\left(-\sin\frac{\pi t}{2}\right)\cdot\frac{d}{dt}\left(\frac{\pi t}{2}\right) \\ &= -\frac{\pi}{2}\sin\frac{\pi t}{2}\cdot\left(\frac{\pi}{2}\right) \\ &= -\frac{\pi^2}{4}\sin\frac{\pi t}{2} \end{aligned}$$

The acceleration after 1 second is

$$a(1) = -\frac{\pi^2}{4}\sin\frac{\pi}{2} = -\frac{\pi^2}{4}\,\frac{\text{feet}}{\text{second}^2}.$$

Part (h)

Below is a plot of the position, velocity, and acceleration versus time for $0 \leq t \leq 6$.



Part (i)

The particle is speeding up when

$$-\frac{\pi^2}{4}\sin\frac{\pi t}{2} > 0$$

$$\sin\frac{\pi t}{2} < 0$$

Note that a sine curve is negative between π and 2π.

$$\pi < \frac{\pi t}{2} < 2\pi$$

$$2 < t < 4.$$

The particle is slowing down when

$$-\frac{\pi^2}{4}\sin\frac{\pi t}{2} < 0$$

$$\sin\frac{\pi t}{2} > 0$$

Note that a sine curve is positive between 0 and π.

$$0 \leq \frac{\pi t}{2} < \pi \quad \text{or} \quad 2\pi < \frac{\pi t}{2} \leq 3\pi$$

$$0 \leq t < 2 \quad \text{or} \quad 4 < t \leq 6.$$